

Universality of small black hole instability in AdS/CFT

Alex Buchel

*Department of Applied Mathematics, Department of Physics and Astronomy,
University of Western Ontario
London, Ontario N6A 5B7, Canada;
Perimeter Institute for Theoretical Physics
Waterloo, Ontario N2J 2W9, Canada*

Abstract

AdS_5 type IIB supergravity compactifications on five-dimensional Einstein manifolds \mathcal{V}_5 realize holographic duals to four-dimensional conformal field theories. Black holes in such geometries are dual to thermal states in these CFTs. When black holes become sufficiently small in (global) AdS_5 , they are expected to suffer Gregory-Laflamme instability with respect to localization on \mathcal{V}_5 . Previously, the instability was demonstrated for gravitational dual of $\mathcal{N} = 4$ SYM, where $\mathcal{V}_5 = S^5$. We extend stability analysis to arbitrary \mathcal{V}_5 . We point out that the quasinormal mode equation governing the instabilities is universal. The precise onset of the instability is \mathcal{V}_5 -sensitive, as it is governed by the lowest non-vanishing eigenvalue λ_{min} of its Laplacian.

September 24, 2015

Contents

1	Introduction	2
2	Stability of AdS_5 black holes smeared on \mathcal{V}_5	3

1 Introduction

Consider type IIB supergravity compactification on a five-dimensional Einstein manifold \mathcal{V}_5 with large five-form flux through it. The vacuum of the resulting five-dimensional effective gravitational action is $\mathcal{M}_5 = AdS_5$, dual to a strongly coupled four-dimensional conformal gauge theory living on the boundary $\partial\mathcal{M}_5$. The best studied realization of the holography is when \mathcal{V}_5 is a five-dimensional sphere, S^5 , in which case the corresponding conformal gauge theory is $\mathcal{N} = 4$ supersymmetric Yang-Mills [1]. Other generalizations include the orbifolds S^5/\mathbb{Z}_k [2], $T^{1,1} = (SU(2) \times SU(2))/U(1)$ [3] and $Y^{p,q}$ Sasaki-Einstein spaces [4]. While the details of the dual CFTs depend on what \mathcal{V}_5 is chosen (*e.g.*, the central charge of the CFT is $\propto \frac{1}{\text{vol}(\mathcal{V}_5)}$), many aspects of the theories are in fact universal. The reason for this commonality stems from the fact that Kaluza-Klein reduction of type IIB supergravity on \mathcal{V}_5 contains a *universal* consistently truncated gravitational sector¹

$$S_5 = \int_{\mathcal{M}_5} d^5\xi \sqrt{-g} (R_5 + 12) . \quad (1.1)$$

We consider the case when $\partial\mathcal{M}_5 = R \times S^3$. Besides (global) AdS_5 vacuum solution, (1.1) contains black holes solutions, dual to thermal states of the boundary CFT. In full ten-dimensional supergravity these black holes are “smeared” on \mathcal{V}_5 . The size of the black hole ρ_+ (as measured by the radius of S^3 at the horizon²) is related to the black hole mass M , compare to the vacuum energy E_{vacuum} as

$$\rho_+^2 = \frac{1}{2} \left(\sqrt{1 + \epsilon} - 1 \right) , \quad \text{where} \quad \epsilon \equiv \frac{M}{E_{vacuum}} . \quad (1.2)$$

Notice that small black holes are light. It is expected that an AdS_5 black hole can not become arbitrarily small: it was proposed in [6, 7] that in the limit $\rho_+ \rightarrow 0$ it would suffer a Gregory-Laflamme (GL) instability [8], resulting in its localization on \mathcal{V}_5 . The latter localization phenomenon was explicitly verified in [5, 9, 10] when $\mathcal{V}_5 = S^5$.

¹Without loss of generality we set the asymptotic AdS_5 radius to unity.

²See [5] for further details and conventions.

One might expect that small AdS_5 black hole localization would depend on details of \mathcal{V}_5 . The purpose of this paper is to explicitly demonstrate that this is not the case — all what matters for determining the onset of the instability is the smallest non-vanishing eigenvalue of the scalar Laplacian on \mathcal{V}_5 , feeding into the fluctuation equation originally constructed by Prestidge in [11], and later obtained for the case $\mathcal{V}_5 = S^5$ in [9].

The rest of the paper is organized as follows. In section 2 we derive a single ”master” second-order quasinormal mode equation in a radial AdS_5 coordinate. Besides ρ_+ , this equation depends parametrically only on the quasinormal mode frequency ω and the scalar Laplacian on \mathcal{V}_5 eigenvalue λ . It is precisely the quasinormal mode equation eq.(5.3) obtained in [5] for the case $\mathcal{V}_5 = S^5$. For the quasinormal mode at the threshold of instability, *i.e.*, for $\omega = 0$, this equation reduces to the threshold equation of Prestidge [11].

2 Stability of AdS_5 black holes smeared on \mathcal{V}_5

Holographic dual to thermal states of a large class of conformal gauge theories on $R \times S^3$ is described by the following type IIB supergravity background

$$\begin{aligned} ds_{10}^2 &= (d\mathcal{M}_5)_{BH}^2 + (d\mathcal{V}_5)^2, & F_5 &= \text{vol}_{\mathcal{M}_5} - \text{vol}_{\mathcal{V}_5}, \\ F_5 &= \star_{10} F_5, & dF_5 &= 0. \end{aligned} \quad (2.1)$$

where $(d\mathcal{M}_5)_{BH}^2$ is the global AdS_5 Schwarzschild black hole metric,

$$\begin{aligned} (d\mathcal{M}_5)_{BH}^2 &= g_{\mu\nu} dx^\mu dx^\nu = -c_1(x)^2 dt^2 + c_2(x)^2 dx^2 + c_3(x)^2 (dS^3)^2, \\ c_1 &= \frac{\sqrt{a(x)}}{\sqrt{x}}, & c_2 &= \frac{1}{2x\sqrt{1-x}\sqrt{a(x)}}, & c_3 &= \frac{\sqrt{1-x}}{\sqrt{x}}, \\ a &= \frac{(x_h + x(1-x_h))(x_h - x)}{x_h^2(1-x)}, & x_h &= \frac{1}{1+\rho_+^2}, & x &\in (0, x_h), \end{aligned} \quad (2.2)$$

and

$$(d\mathcal{V}_5)^2 = g_{\alpha\beta} dy^\alpha dy^\beta. \quad (2.3)$$

$(d\mathcal{V}_5)^2$ is the metric on an Einstein manifold \mathcal{V}_5 , and $(dS^3)^2$ is a round metric on unit radius S^3 . We use μ, ν, ρ, \dots indices on \mathcal{M}_5 , and $\alpha, \beta, \gamma, \dots$ indices on \mathcal{V}_5 .

We are interested in $SO(4)$ -invariant linearized fluctuations of (2.1) that carry an arbitrary angular momentum on \mathcal{V}_5 . Generically, metric fluctuations would couple with

the fluctuations of the 5-form F_5 [5]. Substantial simplification can be achieved with the judicious choice of the gauge. Let

$$\begin{aligned}\delta g_{\mu\nu} &= h_{\mu\nu}(t, x, y^\alpha) \equiv \left\{ \delta g_{tt}, \delta g_{xx}, \delta g_{tx}, \delta g_{ij} \equiv g_{ij}(x) \delta f(t, x, y^\alpha) \right\}, \\ \delta g_{\mu\alpha} &= h_{\mu\alpha}(t, x, y^\beta), \quad \delta g_{\alpha\beta} = h_{\alpha\beta}(t, x, y^\gamma),\end{aligned}\tag{2.4}$$

where we explicitly enumerated non-vanishing components of $\delta g_{\mu\nu}$ consistent with $SO(4)$ symmetry — i, j are angles on S^3 . To leading order in metric fluctuation, the linearized components of the ten-dimensional Ricci tensor on \mathcal{V}_5 take form [12]

$$\begin{aligned}R_{\alpha\beta}^{(1)} &= -\frac{1}{2} \left[\left(\square_{\mathcal{M}_5} + \square_{\mathcal{V}_5} \right) h_{(\alpha\beta)} - 2R_{\alpha\gamma\delta\beta} h^{(\gamma\delta)} - R_\alpha{}^\gamma h_{(\gamma\beta)} - R_\beta{}^\gamma h_{(\gamma\alpha)} \right. \\ &\quad + \frac{1}{5} g_{\alpha\beta} \left(\square_{\mathcal{M}_5} + \square_{\mathcal{V}_5} \right) h_\gamma{}^\gamma - \frac{16}{15} \nabla_\alpha \nabla_\beta h_\gamma{}^\gamma + \nabla_\alpha \nabla_\beta \left(h_\mu{}^\mu + \frac{5}{3} h_\gamma{}^\gamma \right) \\ &\quad \left. - \nabla_\alpha \nabla^\mu h_{\mu\beta} - \nabla_\beta \nabla^\mu h_{\mu\alpha} \right],\end{aligned}\tag{2.5}$$

where

$$h_{\alpha\beta} = h_{(\alpha\beta)} + \frac{1}{5} g_{\alpha\beta} h_\gamma{}^\gamma.\tag{2.6}$$

Assuming that

$$h_\mu{}^\mu = 0, \quad h_{\mu\alpha} = h_{\alpha\beta} = 0,\tag{2.7}$$

we see that

$$R_{\alpha\beta}^{(1)} = 0.\tag{2.8}$$

Notice that with (2.7),

$$\delta \text{vol}_{\mathcal{M}_5} \propto \mathcal{O}(h^2), \quad \delta \text{vol}_{\mathcal{V}_5} = 0,\tag{2.9}$$

which implies that it is consistent, at order $\mathcal{O}(h)$, with the 5-form self-duality constraint and its Bianchi identity not to deform the background 5-form ansatz (2.1). The latter implies that the 5-form stress-energy tensor with components on \mathcal{V}_5 is unchanged from its background value, *i.e.*,

$$T_{\alpha\beta}^{(1)} = 0.\tag{2.10}$$

Together, (2.8) and (2.10) imply that (2.7) solves all Einstein equations with indices on \mathcal{V}_5 .

For linearized components of the Ricci tensor with mixed indices we have [12]

$$R_{\mu\alpha}^{(1)} = -\frac{1}{2} \left[\square_{\mathcal{M}_5} h_{\mu\alpha} - \nabla_\mu \nabla^\nu h_{\nu\alpha} - R_\mu{}^\nu h_{\nu\alpha} + \square_{\mathcal{V}_5} h_{\mu\alpha} - R_\alpha{}^\beta h_{\beta\mu} - \nabla_\alpha \nabla^\nu h_{\nu\mu} + \nabla_\mu \nabla_\alpha (h_\rho{}^\rho + h_\gamma{}^\gamma) - \nabla_\mu \nabla^\beta h_{\beta\alpha} \right]. \quad (2.11)$$

Since in a gauge (2.7) the mixed components of the 5-form stress-energy tensor vanish,

$$T_{\mu\alpha}^{(1)} = 0, \quad (2.12)$$

the Einstein equations with mixed indices reduce to

$$\nabla_\alpha \nabla^\nu h_{\nu\mu} = 0. \quad (2.13)$$

Parameterizing the (traceless) fluctuations as

$$\begin{aligned} \delta g_{tt} &= -c_1(x)^2 e^{-i\omega t} f_1(x) Y_{\mathcal{V}_5}(y^\alpha), & \delta g_{xx} &= c_2(x)^2 e^{-i\omega t} f_2(x) Y_{\mathcal{V}_5}(y^\alpha), \\ \delta g_{tx} &= i e^{-i\omega t} f(x) Y_{\mathcal{V}_5}(y^\alpha), & \delta g_{ij} &= g_{ij}(x) e^{-i\omega t} \left(-\frac{1}{3} f_1(x) - \frac{1}{3} f_2(x) \right) Y_{\mathcal{V}_5}(y^\alpha), \end{aligned} \quad (2.14)$$

equations (2.13) are equivalent to the following two equations

$$\begin{aligned} 0 &= \left(2\omega f_1 c_2 - f \left(\ln \frac{f c_1 c_3^3}{c_2} \right)' \right) \partial_\alpha Y_{\mathcal{V}_5}(y^\gamma) e^{-i\omega t}, \\ 0 &= \left(f_2 (\ln f_2 c_1 c_3^4)' + f_1 \left(\ln \frac{c_3}{c_1} \right)' - \frac{\omega f}{2c_1^2} \right) \partial_\alpha Y_{\mathcal{V}_5}(y^\gamma) e^{-i\omega t}. \end{aligned} \quad (2.15)$$

Remaining Einstein equations³ involve components $_{tt}$, $_{tx}$, $_{xx}$ and a pair of the S^3 components (by $SO(4)$ symmetry):

■ $_{tt}$ components:

$$\begin{aligned} 0 &= \left[\left\{ \frac{c_1^2}{c_2^2} \left(f_1'' + f_1' \left(\ln \frac{c_1 c_3^3}{c_2} \right)' - 2f_2' (\ln c_1)' \right) + \frac{\omega f}{c_2^2} \left(\ln \frac{f c_3^3}{c_2} \right)' - 8f_2 c_1^2 - \omega^2 f_1 \right\} Y_{\mathcal{V}_5} \right. \\ &\quad \left. + f_1 c_1^2 \square_{\mathcal{V}_5} Y_{\mathcal{V}_5} \right] e^{-i\omega t}; \end{aligned} \quad (2.16)$$

■ $_{tx}$ components:

$$0 = i \left[\omega \left\{ f_1 \left(\ln \frac{f_1 c_3}{c_1} \right)' + f_2 \left(\ln \frac{f_2 c_3^4}{c_1} \right)' \right\} Y_{\mathcal{V}_5} + \frac{1}{2} f \square_{\mathcal{V}_5} Y_{\mathcal{V}_5} \right] e^{-i\omega t}; \quad (2.17)$$

³We used here equations of motion for the background (2.1).

■ xx components:

$$0 = \left[\left\{ f_2'' + f_2' \left(\ln \frac{c_1 c_3^5}{c_2} \right)' + 2f_1' \left(\ln \frac{c_3}{c_1} \right)' - \frac{\omega f}{c_1^2} \left(\ln \frac{f}{c_2} \right)' + f_2 c_2^2 \left(8 - \frac{\omega^2}{c_1^2} \right) \right\} Y_{\mathcal{V}_5} - f_2 c_2^2 \square_{\mathcal{V}_5} Y_{\mathcal{V}_5} \right] e^{-i\omega t}; \quad (2.18)$$

■ $S^3 S^3$ components:

$$0 = \left[\frac{1}{3} \left\{ \frac{c_3^2}{c_2^2} \left(f_2'' + f_2' \left(\ln \frac{c_1 c_3^9}{c_2} \right)' + f_1'' + f_1' \left(\ln \frac{c_1 c_3^3}{c_2} \right)' \right) + \frac{2}{c_2^2} (c_3^2)' (\ln c_1 c_3)' (f_1 + 4f_2) - \frac{3\omega f (c_3^2)'}{2c_1^2 c_2^2} - (f_1 + f_2) c_3^2 \left(8 - \frac{\omega^2}{c_1^2} \right) \right\} Y_{\mathcal{V}_5} + \left(\frac{1}{3} f_1 + \frac{1}{3} f_2 \right) c_3^2 \square_{\mathcal{V}_5} Y_{\mathcal{V}_5} \right] e^{-i\omega t}. \quad (2.19)$$

When $Y_{\mathcal{V}_5}$ is an eigenfunction on \mathcal{V}_5 , *i.e.*,

$$\square_{\mathcal{V}_5} Y_{\mathcal{V}_5} = -\lambda Y_{\mathcal{V}_5}, \quad (2.20)$$

the PDEs (2.15)-(2.19) reduce to a system of ODEs in AdS_5 radial coordinate x . The resulting system of ODEs is over-determined, which can be exploited to reduce it to the following equations:

$$\begin{aligned} 0 &= 2c_3\omega \left(c_2^2 c_1^2 \lambda + 4c_2^2 c_1^2 - c_2^2 \omega^2 - (c_1')^2 \right) f_2 + 2\omega \left(4c_2^2 c_1^2 c_3 - c_2^2 c_3 \omega^2 - 3c_1 c_1' c_3' \right. \\ &\quad \left. - c_3 (c_1')^2 \right) f_1 + \left(c_1^2 c_3' \lambda - c_1 c_3 c_1' \lambda + 3c_3' \omega^2 \right) f, \\ 0 &= f' + f \left(\ln \frac{c_1 c_3^3}{c_2} \right)' - 2\omega c_2^2 f_1, \\ 0 &= f_1' + \left(\frac{\omega}{2c_1^2} - \frac{\lambda}{2\omega} \right) f - 2(\ln c_1)' f_2. \end{aligned} \quad (2.21)$$

Remarkably, a solution to (2.21), together with (2.20), solves (2.15)-(2.19).

Using the first two equations we can algebraically eliminate f_1 and f_2 in favor of $\{f, f'\}$. The third equation in (2.21) produces the "master" quasinormal mode

equation:

$$\begin{aligned}
0 = & f'' + f' \left(3(\rho_+^2 + 1)^3 y^{14} - (5\rho_+^4 + 5\rho_+^2 + 9)(\rho_+^2 + 1)^2 y^{12} - \rho_+^2(\rho_+^2 + 1)^2(\lambda + 22)y^{10} \right. \\
& - \rho_+^2(\rho_+^2 + 1)(9\lambda\rho_+^4 + 61\rho_+^4 + 9\lambda\rho_+^2 + 3\omega^2 + 61\rho_+^2 + 2\lambda + 18)y^8 + \rho_+^2(-11\omega^2\rho_+^4 \\
& + 8\lambda\rho_+^4 - 11\omega^2\rho_+^2 + 11\rho_+^4 + 8\lambda\rho_+^2 - 3\omega^2 + 11\rho_+^2 + 3\lambda + 12)y^6 + \rho_+^4(10\lambda\rho_+^4 + 21\rho_+^4 \\
& + 10\lambda\rho_+^2 - 4\omega^2 + 21\rho_+^2 + 5\lambda + 19)y^4 + \rho_+^6(-\omega^2 + \lambda + 4)y^2 - \rho_+^8(\lambda + 3) \Big) \Big(\\
& y(y^2 - 1)(\rho_+^2 + y^2)((\rho_+^2 + 1)y^2 + \rho_+^2)(-\rho_+^2 + 1)^2 y^8 - \rho_+^2(\rho_+^2 + 1)(\lambda + 6)y^4 \\
& + \rho_+^2(-\omega^2 + \lambda + 4)y^2 + \rho_+^4(\lambda + 3)) \Big)^{-1} + \left(4(\rho_+^2 + 1)^4 y^{20} \right. \\
& - (8(2\rho_+^4 + 2\rho_+^2 + 3))(\rho_+^2 + 1)^3 y^{18} - (\rho_+^2 + 1)^2(-4\rho_+^8 - 8\rho_+^6 + 7\lambda\rho_+^4 + 44\rho_+^4 + 7\lambda\rho_+^2 \\
& + 48\rho_+^2 - 24)y^{16} - \rho_+^2(\rho_+^2 + 1)^2(22\lambda\rho_+^4 + 96\rho_+^4 + 22\lambda\rho_+^2 + \omega^2 + 96\rho_+^2 - 13\lambda - 112)y^{14} \\
& + \rho_+^2(\rho_+^2 + 1)(9\lambda\rho_+^8 + 72\rho_+^8 + 18\lambda\rho_+^6 + \lambda^2\rho_+^4 - 2\omega^2\rho_+^4 + 144\rho_+^6 + 76\lambda\rho_+^4 + \lambda^2\rho_+^2 \\
& - 2\omega^2\rho_+^2 + 416\rho_+^4 + 67\lambda\rho_+^2 + 6\omega^2 + 344\rho_+^2 - 6\lambda)y^{12} + \rho_+^4(\rho_+^2 + 1)(2\lambda^2\rho_+^4 + 23\omega^2\rho_+^4 \\
& + 39\lambda\rho_+^4 + 2\lambda^2\rho_+^2 + 23\omega^2\rho_+^2 + 184\rho_+^4 + 2\lambda\omega^2 + 39\lambda\rho_+^2 - 2\lambda^2 + 42\omega^2 + 184\rho_+^2 - 58\lambda \\
& - 120)y^{10} + \rho_+^4(\lambda^2\rho_+^8 - 15\lambda\rho_+^8 + 2\lambda^2\rho_+^6 - 48\rho_+^8 + 4\lambda\omega^2\rho_+^4 - 30\lambda\rho_+^6 - 5\lambda^2\rho_+^4 + 42\omega^2\rho_+^4 \\
& - 96\rho_+^6 + 4\lambda\omega^2\rho_+^2 - 114\lambda\rho_+^4 - 6\lambda^2\rho_+^2 + \omega^4 + 42\omega^2\rho_+^2 - 240\rho_+^4 - 2\lambda\omega^2 - 99\lambda\rho_+^2 + \lambda^2 \\
& - 14\omega^2 - 192\rho_+^2 + 14\lambda + 40)y^8 - \rho_+^6(-2\lambda\omega^2\rho_+^4 + 6\lambda^2\rho_+^4 - 6\omega^2\rho_+^4 - 2\lambda\omega^2\rho_+^2 + 48\lambda\rho_+^4 \\
& + 6\lambda^2\rho_+^2 - 2\omega^4 - 6\omega^2\rho_+^2 + 48\rho_+^4 + 6\lambda\omega^2 + 48\lambda\rho_+^2 - 4\lambda^2 + 37\omega^2 + 48\rho_+^2 - 49\lambda - 128)y^6 \\
& - \rho_+^8(2\lambda^2\rho_+^4 + \lambda\rho_+^4 + 2\lambda^2\rho_+^2 - \omega^4 - 24\rho_+^4 + 6\lambda\omega^2 + \lambda\rho_+^2 - 6\lambda^2 + 32\omega^2 - 24\rho_+^2 - 63\lambda \\
& - 148)y^4 + \rho_+^{10}(-2\lambda\omega^2 + 4\lambda^2 - 9\omega^2 + 35\lambda + 72)y^2 + \rho_+^{12}(\lambda + 4)(\lambda + 3) \Big) \Big(\\
& (y^2 - 1)^2((\rho_+^2 + 1)^2 y^8 + \rho_+^2(\rho_+^2 + 1)(\lambda + 6)y^4 - \rho_+^2(-\omega^2 + \lambda + 4)y^2 \\
& - \rho_+^4(\lambda + 3))((\rho_+^2 + 1)y^2 + \rho_+^2)^2(\rho_+^2 + y^2)^2 y^2 \Big)^{-1} f,
\end{aligned} \tag{2.22}$$

where we introduced a new radial coordinate y , so that

$$x \equiv \frac{y^2}{y^2 + \rho_+^2}, \quad y \in (0, 1). \tag{2.23}$$

This is our universal quasinormal mode equation: the only information about \mathcal{V}_5 is in the choice of the scalar Laplacian eigenvalue λ .

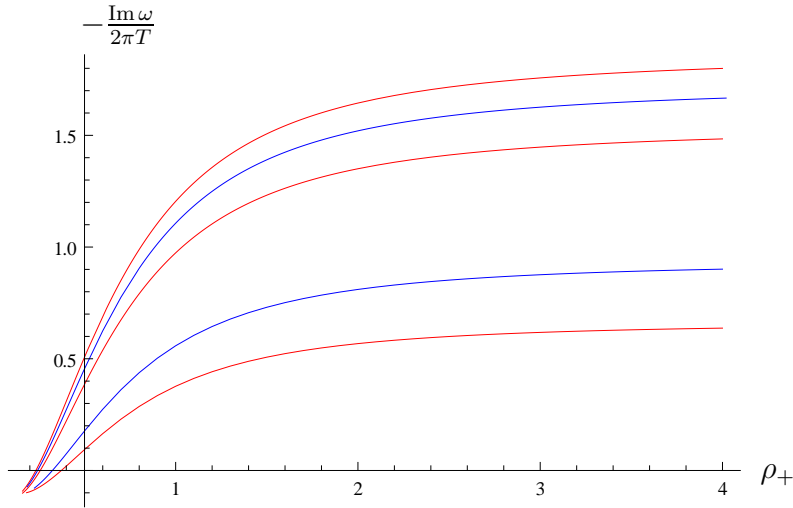


Figure 1: The dependence of $g = -\text{Im}(\omega)$ as a function of a black hole size ρ_+ and temperature $T = \frac{2\rho_+^2+1}{2\pi\rho_+}$ in KW model for $U(1)_R$ charged/neutral quasinormal modes (red/blue) with $T^{1,1}$ eigenvalues $\lambda_{j,\ell,r}$: $(j, \ell, r) = \{(\frac{1}{2}, \frac{1}{2}, 1), (1, 1, 2), (\frac{3}{2}, \frac{1}{2}, 1), (1, 0, 0), (1, 1, 0)\}$. g increases with $\lambda_{j,\ell,r}$. Black holes with $g < 0$ are unstable with respect to condensation of these fluctuations.

Equation (2.22) is identical to eq.(5.3) derived in [5], provided we identify⁴

$$f_{xy} \implies \frac{y}{y^2 + \rho_+^2} f, \quad s \implies \lambda. \quad (2.24)$$

Additionally, when $\omega = 0$ it reduces to the equation at the threshold of instability, originally derived in [11].

We now consider a simple application of (2.22) in the context of the holographic Klebanov-Witten (KW) model [3]. In this case \mathcal{V}_5 is $T^{1,1}$ coset manifold. Properties of the Laplacian on $T^{1,1}$ were extensively studied in [13–15]. The eigenvalues are completely determined by a pair of $SU(2)$ spins $\{j, \ell\} \in \frac{1}{2}\mathbb{Z}$ and a $U(1)_R$ R -symmetry charge $r \in \mathbb{Z}$ as follows

$$\lambda = \lambda_{j,\ell,r} = 6 \left(j(j+1) + \ell(\ell+1) - \frac{1}{8}r^2 \right). \quad (2.25)$$

A triplet $\{j, \ell, r\}$ is constraint so that both $2j$ and 2ℓ have the same parity, and

$$r \leq \min\{2j, 2\ell\}. \quad (2.26)$$

⁴We refer the reader to [5] for the details associated with solving (2.22).

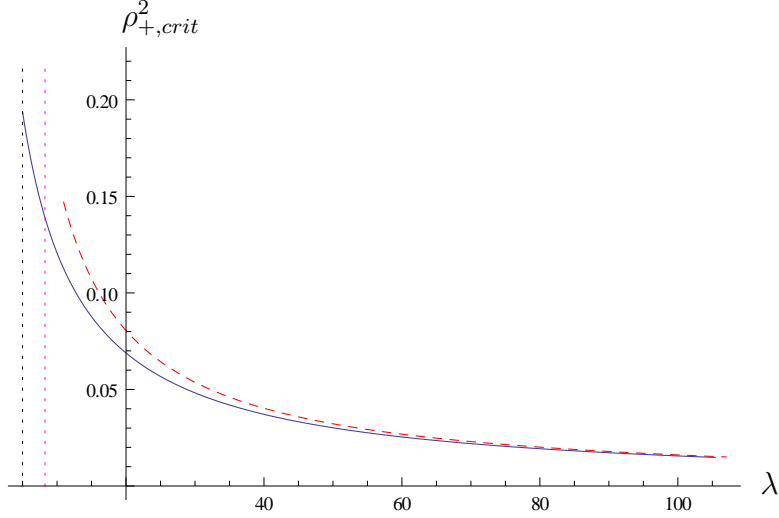


Figure 2: Critical size of the AdS_5 black hole $\rho_{+,crit}^2$ below which a quasinormal mode with an eigenvalue λ on \mathcal{V}_5 becomes unstable. The dashed red line is a large- λ asymptotic (2.28). The black and the magenta vertical dotted lines correspond to the onset of the instability (the smallest non-vanishing value of λ on \mathcal{V}_5) for $\mathcal{V}_5 = S^5$ and $\mathcal{V}_5 = T^{1,1}$ correspondingly.

Note that the lowest non-vanishing eigenvalue on $T^{1,1}$ is

$$\lambda_{min} = \lambda_{\frac{1}{2}, \frac{1}{2}, 1} = \frac{33}{4}. \quad (2.27)$$

The spectrum of the low-lying quasinormal modes of AdS_5 black holes in KW holography is shown on figure 1. The red curves correspond to states carrying $U(1)_R$ charge, and the blue curves represent neutral states.

The solid blue curve in figure 2 presents the threshold value $\rho_{+,crit}^2$ of the Gregory-Laflamme instability, corresponding to $\omega = 0$, for the $AdS_5 \times \mathcal{V}_5$ quasinormal mode with the \mathcal{V}_5 eigenvalue λ . The dashed red line is the large- λ asymptotic, see [5],

$$\rho_{+,crit}^2 = \frac{1.61015}{\lambda} + \mathcal{O}(\lambda^{-2}). \quad (2.28)$$

Notice that larger values of λ result in smaller threshold values of $\rho_{+,crit}^2$. Thus, the onset of the instability of smeared $AdS_5 \times \mathcal{V}_5$ black holes is determined by the smallest non-vanishing eigenvalue λ of the scalar Laplacian on \mathcal{V}_5 .

Acknowledgments

Research at Perimeter Institute is supported by the Government of Canada through Industry Canada and by the Province of Ontario through the Ministry of Research & Innovation. This work was further supported by NSERC through the Discovery Grants program.

References

- [1] J. M. Maldacena, “The large N limit of superconformal field theories and supergravity,” *Adv. Theor. Math. Phys.* **2**, 231 (1998) [*Int. J. Theor. Phys.* **38**, 1113 (1999)] [arXiv:hep-th/9711200].
- [2] S. Kachru and E. Silverstein, “4-D conformal theories and strings on orbifolds,” *Phys. Rev. Lett.* **80**, 4855 (1998) [hep-th/9802183].
- [3] I. R. Klebanov and E. Witten, “AdS / CFT correspondence and symmetry breaking,” *Nucl. Phys. B* **556**, 89 (1999) [hep-th/9905104].
- [4] J. P. Gauntlett, D. Martelli, J. Sparks and D. Waldram, *Adv. Theor. Math. Phys.* **8**, 711 (2004) [hep-th/0403002].
- [5] A. Buchel and L. Lehner, “Small black holes in $AdS_5 \times S^5$,” *Class. Quant. Grav.* **32**, no. 14, 145003 (2015) [arXiv:1502.01574 [hep-th]].
- [6] T. Banks, M. R. Douglas, G. T. Horowitz and E. J. Martinec, “AdS dynamics from conformal field theory,” hep-th/9808016.
- [7] G. T. Horowitz, “Comments on black holes in string theory,” *Class. Quant. Grav.* **17**, 1107 (2000) [hep-th/9910082].
- [8] R. Gregory and R. Laflamme, “Black strings and p-branes are unstable,” *Phys. Rev. Lett.* **70**, 2837 (1993) [hep-th/9301052].
- [9] V. E. Hubeny and M. Rangamani, “Unstable horizons,” *JHEP* **0205**, 027 (2002) [hep-th/0202189].
- [10] O. J. C. Dias, J. E. Santos and B. Way, “Lumpy $AdS_5 \times S^5$ black holes and black belts,” *JHEP* **1504**, 060 (2015) [arXiv:1501.06574 [hep-th]].

- [11] T. Prestidge, “Dynamic and thermodynamic stability and negative modes in Schwarzschild-anti-de Sitter,” *Phys. Rev. D* **61**, 084002 (2000) [hep-th/9907163].
- [12] O. DeWolfe, D. Z. Freedman, S. S. Gubser, G. T. Horowitz and I. Mitra, “Stability of $\text{AdS}(p) \times \text{M}(q)$ compactifications without supersymmetry,” *Phys. Rev. D* **65**, 064033 (2002) [hep-th/0105047].
- [13] S. S. Gubser, “Einstein manifolds and conformal field theories,” *Phys. Rev. D* **59**, 025006 (1999) [hep-th/9807164].
- [14] A. Ceresole, G. Dall’Agata and R. D’Auria, “K K spectroscopy of type IIB supergravity on $\text{AdS}(5) \times T^{**}11$,” *JHEP* **9911**, 009 (1999) [hep-th/9907216].
- [15] A. Ceresole, G. Dall’Agata, R. D’Auria and S. Ferrara, “Superconformal field theories from IIB spectroscopy on $\text{AdS}(5) \times T^{**}11$,” *Class. Quant. Grav.* **17**, 1017 (2000) [hep-th/9910066].